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10MAT31

Third Semester B.E. Degree Examination, December 2012
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Find the Fourier series of $f(x) = x - x^2$, $-\pi \le x \le \pi$. Hence deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots\ldots\ldots\ldots = \frac{\pi^2}{12}$$

Is the above deduced series convergent? (Answer in Yes or No) **(07 Marks)**

b. Define : i) Half range Fourier sine series of f(x)
ii) Complex form of Fourier series of f(x)
Find the half range cosine series of $f(x) = x$ in $0 < x < 2$. **(07 Marks)**

c. Obtain a_0 , a_1 , b_1 in the Fourier expansion of y , using harmonic analysis for the data given.

x	0	1	2	3	4	5
y	9	18	24	28	26	20

(06 Marks)

2 a. Find the Fourier transform of

$$f(x) = 1 - x^2 \quad \text{for} \quad |x| \le 1$$
$$= 0 \quad \text{for} \quad |x| > 1$$

Hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3}\cos\left(\frac{x}{2}\right)dx$ **(07 Marks)**

b. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$ **(07 Marks)**

c. Find the Fourier cosine transform of

$$f(x) = 4x \quad , \quad \text{for} \quad 0 < x < 1$$
$$= 4 - x \quad , \quad \text{for} \quad 1 < x < 4$$
$$= 0 \quad , \quad \text{for} \quad x > 4$$

(06 Marks)

3 a. i) Write down the two dimensional heat flow equation (p d e) in steady state (or two dimensional) Laplace's equation. Just mention.
ii) Solve one dimensional heat equation by the method of separation of variables. **(07 Marks)**

b. Using D'Alembert's method, solve one dimensional wave equation. **(07 Marks)**

c. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form of $y = a \sin(\pi x/l)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is,

$$y(x,t) = a\sin\left(\frac{\pi x}{\ell}\right)\cos\left(\frac{\pi ct}{\ell}\right)$$

Start the answer assuming the solution to be

$$y = (C_1\cos(px) + C_2\sin(px))(C_3\cos(cpt) + C_4\sin(cpt))$$

(06 Marks)

4 a. Fit a linear law, P = mW + C, using the data

P	12	15	21	25
W	50	70	100	120

(06 Marks)

b. Find the best values of a and b by fitting the law $V = at^b$ using method of least squares for the data,

V (ft/min)	350	400	500	600
t (min)	61	26	7	26

Use base 10 for algorithm for computation. **(07 Marks)**

c. Using simplex method,
Maximize $Z = 5x_1 + 3x_2$
Subject to, $x_1 + x_2 \le 2$; $5x_1 + 2x_2 \le 10$; $3x_1 + 8x_2 \le 12$; $x_1, x_2 \ge 0$. **(07 Marks)**

PART – B

5 a. Use Newton-Raphson method, to find the real root of the equation $3x = (\cos x) + 1$.
Take $x_0 = 0.6$. Perform two iterations. **(06 Marks)**

b. Apply Gauss-Seidel iteration method to solve equations

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3z + 20z = 25$$

Assume initial approximation to be $x = y = z = 0$. Perform three iterations. **(07 Marks)**

c. Using Rayleigh's power method to find the largest eigen value and the corresponding eigen vector of the matrix.

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Take $[1 \ 0 \ 0]^T$ as the initial approximation. Perform four iterations. **(07 Marks)**

6 a. Use appropriate interpolating formula to compute y(82) and y(98) for the data

x	80	85	90	95	100
y	5026	5674	6362	7088	7854

(07 Marks)

b. i) For the points (x_0, y_0) (x_1, y_1) (x_2, y_2) mention Lagrage's interpolation formula.
ii) If f(1) = 4, f(3) = 32, f(4) = 55, f(6) = 119 ; find interpolating polynomial by Newton's divided difference formula. **(07 Marks)**

c. Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$, using

i) Simpson's 1/3rd rule ii) Simpson's 3/8th rule iii) Weddele's rule, using

x	0	1	2	3	4	5	6
$f(x) = \frac{1}{1+x^2}$	1	0.5	0.2	0.4	0.0588	0.0385	0.027

(06 Marks)

7 a. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to u(0, t), u(4, t) = 0. $u_t(x, 0) = 0$ and u(x, 0) = x(4 - x) by taking h = 1, k = 0.5 upto four steps. **(07 Marks)**

b. Solve two dimensional Laplace equation at the pivotal or nodal points of the mesh shown in Fig.Q7(b). To find the initial values assume $u_4 = 0$. Perform three iterations including computation of initial values. **(07 Marks)**

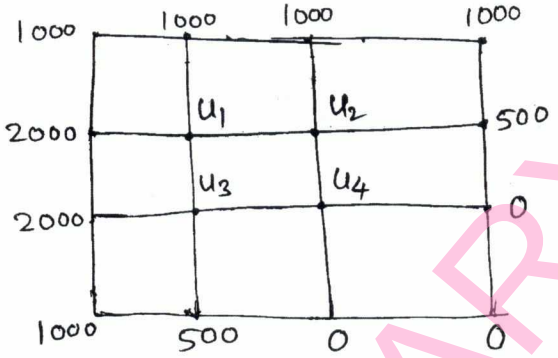

Fig.Q7(b)

c. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to the conditions u(x, o) = sinπx, o ≤ x ≤ 1; u(0, t) = u(1, t) = 0. Carry out computations for two levels, taking h = 1/3 , k = 1/36. **(06 Marks)**

8 a. Find the z-transform of

$$\frac{n}{3^n} + 2^n n^2 + 4\cos(n\theta) + 4^n + 8$$ **(07 Marks)**

b. State and prove i) Initial value theorem ii) Final value theorem of z-transforms. **(07 Marks)**

c. Using the z-transform solve

$u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0$, $u_1 = 1$. **(06 Marks)**

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10CV32

Third Semester B.E. Degree Examination, December 2012
Building Materials and Construction Technology

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Explain in detail the plate load test for determining safe bearing capacity of soil. **(08 Marks)**

b. Enumerate various methods of sub soil exploration. What are the factors on which the choice of a particular method depends? **(06 Marks)**

c. Mention the situations in which the pile foundations are adopted and also explain the classification of pile foundation. **(06 Marks)**

2 a. Sketch the elevation of a brickwall built in i) English bond ii) Flemish bond. Compare the merits and demerits of English bond and Flemish bond. **(10 Marks)**

b. Explain with sketches, various types of joints used in stone masonry. **(10 Marks)**

3 a. Draw a neat sketch an arch and label various technical terms used in the construction. **(10 Marks)**

b. Classify various types of lintels and discuss their respective uses. **(10 Marks)**

4 a. Sketch the elevation of wooden queen post roof truss. Label the different parts. **(10 Marks)**

b. Write short notes on the following types of flooring:
i) Asphalt flooring ii) Linoleum flooring iii) P.V.C. flooring
iv) Cork flooring v) Rubber flooring. **(10 Marks)**

PART – B

5 a. What are the factors considered while locating doors and windows? **(06 Marks)**

b. Draw a labeled sketch of battened ledged and braced door. **(08 Marks)**

c. Write notes on the following:
i) Revolving door ii) Louvered window iii) Clear-storey window **(06 Marks)**

6 a. State briefly the requirements of a good stair. **(06 Marks)**

b. Explain with the help of sketches, the following terms:
i) Landing ii) Noising iii) Going iv) Stringer v) Newel post vi) Handrail **(08 Marks)**

c. Differentiate between
i) Helical stair and spiral stair
ii) R.C.C. stair with slab spanning horizontally and slab spanning longitudinally. **(06 Marks)**

7 a. Explain the purpose of plastering. Explain the various types of mortars used for plastering. **(08 Marks)**

b. What is the purpose of painting? **(04 Marks)**

c. Mention the characteristics of an ideal paint. **(08 Marks)**

8 a. What do you understand by the term shoring? Under what circumstances is it necessary? With neat sketch explain the raking shore. **(08 Marks)**

b. Write short notes on : i) Scaffolding ii) Underpinning. **(06 Marks)**

c. What are the effects of dampness? Mention the methods of damp proofing. **(06 Marks)**

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10CV/EV33

Third Semester B.E. Degree Examination, December 2012
Strength of Materials

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART - A

1 a. A rod of 12mm diameter and 1m long subjected to a tensile load "P" such way that elongation should not be more than 0.4mm. Find the value of 'P'. Take $E = 2 \times 10^5 N/mm^2$. **(05 Marks)**

b. Derive an equation for deformation in tapering circular bar subjected to an axial load P. **(06 Marks)**

c. The bar shown in fig.Q1(c) is tested in universal testing machine. It is observed that at a load of 40kN the total extension is 0.285mm. Determine the Young's modulus of the material. **(09 Marks)**

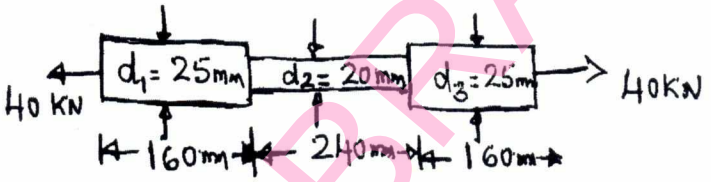


Fig.Q1(c)

2 a. Derive the relation between modulus of rigidity Young's modulus and Poisson ratio [E, C and U]. **(06 Marks)**

b. A rod is 10m long at 10^0C. Find the expansion of the bar, when the temperature rises to 80^0C. If the expansion is prevented, determine the temperature stress developed. Take $E = 1 \times 10^5 N/mm^2$; $\alpha = 12 \times 10^{-6}/^0C$. **(06 Marks)**

c. A 12mm diameter specimen is subjected to a tensile force of 20kN and deformation is 0.3mm, observed over a gauge length of 150mm. The reduction in diameter is 0.0079mm. Determine the Elastic constants. **(08 Marks)**

3 a. What are the principal stresses and principal planes? **(04 Marks)**

b. Explain the construction of Mohr's circle for compound stress in two – dimensional system. **(06 Marks)**

c. The state of stress at a point in a strained material is as shown in fig. Q3(c). Determine
i) Direction of the principal planes ii) Magnitude of principal stresses
iii) Magnitude of maximum shear stress and direction. **(10 Marks)**

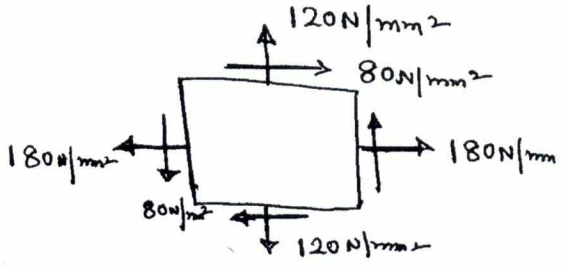
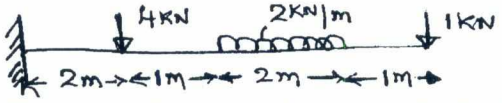


Fig.Q3(c)

4 a. Explain the terms : i) Hogging bending moment ii) Sagging bending moment
iii) Points of contraflexture. **(05 Marks)**

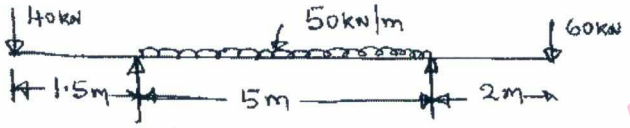
b. For the cantilever beam shown in fig.Q4(b), obtain the shear force and the bending moment diagram. **(05 Marks)**

Fig.Q4(b)



c. Draw the SFD and BMD for the beam as shown in fig.Q4(c). Indicate the maximum bending moment and its location. Also indicate point of contraflexture. **(10 Marks)**

Fig.Q4(c)



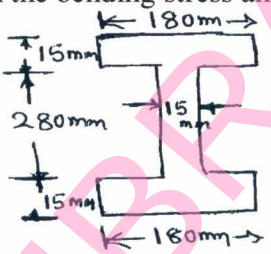
PART - B

5 a. Explain the terms : i) Neutral axis ii) Section modulus iii) Moment of resistance. **(06 Marks)**

b. State the assumptions made in pure bending theory. **(04 Marks)**

c. A beam with I section as shown in fig. Q.5(c), is subjected to a bending moment 120kN – m and a shear force of 60kN. Sketch the bending stress and shear stress diagram. **(10 Marks)**

Fig.Q5(c)



6 a. Explain the terms : i) Slope ii) Deflection iii) Deflection curve. **(06 Marks)**

b. A simply supported beam of span 'L' m carries a central point of 'W' at its centre. Determine maximum slope and deflection. **(04 Marks)**

c. A simply supported beam of length 6m is simply supported at its ends and carries two point loads of 48kN and 40kN at a distance of 1m and 3m respectively from the left support. Find deflection under each load. Take $E = 2 \times 10^5 N/mm^2$ $I = 8.5 \times 10^7 mm^4$. **(10 Marks)**

7 a. Derive the torsion equation for circular member $\frac{T}{I_p} = \frac{F_s}{R} = \frac{G_\theta}{L}$, with usual notations. **(10 Marks)**

b. A solid circular shaft is to transmit 250kW at 100 rpm. If the shear stress is not to exceeded $75N/mm^2$, what should be the diameter of the shaft? If this shaft is to be replaced by a hollow one whose internal diameter is 0.6 times external diameter. Determine the size and percentage saving in weight, maximum shear stress being the same. **(10 Marks)**

8 a. Explain the terms : i) Buckling load ii) Slenderness ratio iii) Effective length of columns. **(06 Marks)**

b. Derive the Euler's expression for buckling load for columns with both ends hinged. **(04 Marks)**

c. A solid round bar 4m long and 50mm in diameter was found to extend by 4.6mm long under a tensile load of 50kN. This bar is used a strut with both ends hinged [Pinned]. Determine Euler's crippling load for the bar and also safe load taking factor of safety as 4. **(10 Marks)**

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10CV/EV34

Third Semester B.E. Degree Examination, December 2012
Surveying – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. What is surveying? Explain the basic principle of surveying. **(06 Marks)**

b. Differentiate between plan and map. **(04 Marks)**

c. Give the broad classification of surveying. **(10 Marks)**

2 a. What are the different types of tapes and chains used in surveying? **(06 Marks)**

b. Explain indirect method of ranging with sketch. **(04 Marks)**

c. A 30 m chain was tested before commencement of the day's work and found to be correct. After chaining 100 chains the chain was found to be 5 cm too long. At the end of day's work, after chaining a total distance of 180 chains, the chain was found to be 10 cm too long. What is the true distance chained? **(10 Marks)**

3 a. Explain the working principle of optical square. **(05 Marks)**

b. What are the factors to be considered for the selection of a station? **(05 Marks)**

c. There is an obstacle in the form of a pond on the main line AB. Two points C and D were taken on the opposite sides of the pond. On the left of CD, a line CE was laid out 120m in length and a second line CF 80 m was laid on the right of CD such that E, D and F are in a line. Determine the obstructed length CD. Give ED = 180 m and DF = 165m. **(10 Marks)**

4 a. Distinguish between:
i) Magnetic bearing and true bearing
ii) Whole circle bearing and reduced bearing. **(06 Marks)**

b. The following bearings were observed with a prismatic compass. Calculate interior angles. Apply check. **(14 Marks)**

Line	Fore bearing
AB	64° 30′
BC	130° 0′
CD	47° 0′
DE	210° 30′
EA	310° 30′

PART – B

5 a. What is local attaraction? Explain methods to correct the local attaraction. **(08 Marks)**

b. The following bearings were observed in traveling with a prismatic compass in a place where local attaraction was suspected. Compute the correct bearings of the lines. **(12 Marks)**

Line	Fore bearing	Back bearing
AB	38° 30′	219° 15′
BC	100° 45′	278° 30′
CD	25° 45′	207° 30′
DE	325° 15′	145° 15′
EA	190° 30′	10° 15′

6 a. What are the temporary adjustments of a dumpy level? Explain. **(06 Marks)**

b. Enlist types of leveling. **(04 Marks)**

c. Following observations were taken in reciprocal levelling:

Instrument at	Staff readings on		Remarks
	A	B	
A	1.545 m	2.565 m	Dist AB = 1420 m
	0.725m	1.935 m	RL of A = 108.360 m

i) Find the reduced level of B (True RL)

ii) Combined correction for curvature and refraction. **(10 Marks)**

7 a. What is contour? Explain characteristics of contours. **(06 Marks)**

b. The following readings were observed successively with a leveling instrument. The instrument was shifted after 5th and 11th readings.

0.585, 1.010, 1.735, 3.295, 3.755, 0.350, 1.300, 1.795, 2.575, 3.375, 3.895, 1.735, 0.635, 1.605.

Rule out a page of level book and determine the RL of various points if RL of first point is 136.440 m using rise and fall method. **(14 Marks)**

8 a. What are the different methods of plane table surveying? **(08 Marks)**

b. With a neat sketch, explain the solution to three point problem. **(12 Marks)**

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10CV35

Third Semester B.E. Degree Examination, January 2013
Fluid Mechanics

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Define the following terms, mention their units.
   i) Mass density
   ii) Specific weight
   iii) Specific volume. **(06 Marks)**

b. A glass tube 2.5 mm in diameter contains mercury column with air above it. If σ =0.50 N/m, What will be the capillary depression? Take $\theta = 135°$. **(04 Marks)**

c. Two large surfaces are 2.5 cm apart. This space is filled with an oil of viscosity 0.80 Ns/m^2. Find the force required to drag a thin plate of area 0.5 m^2 between the two surfaces at a speed of 0.6 m/s.
   i) When the plate is equidistant from the surfaces
   ii) When the plate is at 1 cm from one of the surfaces. **(10 Marks)**

2 a. Distinguish between simple manometer and differential manometer, with the help of sketches. **(06 Marks)**

b. Explain the different types of pressure. Give the relationship between them. Indicate these pressures by means of a sketch. **(06 Marks)**

c. Petrol of specific gravity 0.8 flows up through a vertical pipe. A and B are the two points in the pipe, B being 0.3 m higher than A. Connections are led from A and B to a U tube manometer containing mercury. If the pressure difference between A and B is 18 KPa, find the reading of differential manometer. **(08 Marks)**

3 a. Define :
   i) Total pressure
   ii) Centre of pressure. **(04 Marks)**

b. Derive an expression fro total pressure and centre of pressure for an inclined plane surface immersed in a liquid of specific weight γ. **(08 Marks)**

c. A rectangular plate 2m wide and 3m depth is immersed in water such that its ends are at depths of 1.5 m and 3m respectively. Determine the total pressure acting on the plate and locate centre of pressure. **(08 Marks)**

4 a. Explain the classification of flow based on Reynold's number. **(04 Marks)**

b. Derive the continuity equation in Cartesian coordinates for steady, incompressible, three dimensional flows. **(08 Marks)**

c. The velocity components in a two dimensional flow fields is given by $u = y^3/3 + 2x - x^2y$, $v = xy^2 - 2y - x^3/3$. Show that these functions represent the conditions for an irrotational flow. Obtain an expression for stream function. **(08 Marks)**

PART – B

5 a. Derive Bernoulli's equation of energy along a stream line. State the assumptions and limitations made in deriving the equation. **(10 Marks)**

b. The following are the date given for laying a water supply pipe line. The change in diameter is gradual from 20 cm at A to 50 cm at B. pressure at A and B are 80 kN/m^2 and 60 kN/m^2 respectively. The end B is 3 m higher than A. If the flow in the pipe is 200 lit/s, find :

i) The direction of flow

ii) The loss of head due to friction between A and B. **(10 Marks)**

6 a. Explain :

i) Pipes in series

ii) Phenomenon of water hammer in pipes. **(06 Marks)**

b. Explain minor losses. Give expression for head loss due to

i) Sudden enlargement

ii) Major loss. **(06 Marks)**

c. A compound piping system consists of 1800 m of 0.5 m, 1200 m of 0.4 m and 600 m of 0.3 m new cast iron pipes connected in series. Convert the system to

i) An equivalent length of 0.4 m pipe

ii) Equivalent size pipe 3600 m long. **(08 Marks)**

7 a. Explain with the help of a neat sketch, the working of cup type currentmeter. Give the equation for finding the velocity of flow using currentmeter. **(08 Marks)**

b. Distinguish between :

i) Weight gauge and float gauge

ii) Point gauge and hook gauge

iii) Self record up gauge and staff gauge

iv) Surface velocity and mean velocity. **(08 Marks)**

c. A pilot tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6 m and static pressure head is 5 m. Calculate the velocity of flow assuming the coefficient of pilot tube = 0.98. **(04 Marks)**

8 a. Explain cipolletti notch. What is the advantage of cipolletti notch over trapezoidal notch? Give the equation of discharge over a cipolletti notch. **(06 Marks)**

b. Distinguish between :

i) Notch and weir

ii) Venturimeter and orificemeter

iii) Coefficient of velocity and coefficient of discharge. **(06 Marks)**

c. Find the discharge over 10 m long rectangular weir under a head if 2 m, if the channel approaching the weir is 20 m wide and 2.5 m deep. Consider velocity of approach. Assume $C_d = 0.6$. Neglect end contraction. Take one trial. **(08 Marks)**

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10CV36

Third Semester B.E. Degree Examination, January 2013
Applied Engineering Geology

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Discuss in brief, the importance of geological knowledge to the civil engineering. **(05 Marks)**
b. With neat sketch, name the different part of the earth interior and explain them. **(05 Marks)**
c. Discuss the following physical properties of minerals HARDNESS, CLEAVAGE, STREAK. **(06 Marks)**
d. Write the chemical composition and uses of any two of the following minerals GALENA, CALCITE, ORTHO CLASE FELDSPAR QUARTZ. **(04 Marks)**

2 a. What is extrusive and intrusive forms of igneous rocks? Explain any two of them, with neat diagram. **(06 Marks)**
b. Write the classification of sedimentary rocks based on grain size of sediments. Give one example of rock to each type. **(06 Marks)**
c. What is metamorphism? Describe any two types of metamorphism. **(04 Marks)**
d. Write the properties and uses of any two of the following rocks GRANITE, BASALT, SAND STONE, MARBLE. **(04 Marks)**

3 a. Write a brief note on Epigene and Hypogene geological agents. **(04 Marks)**
b. What is mechanical weathering? Write a note on mechanical weathering by thermal effect. **(05 Marks)**
c. What is soil erosion? Write a note on soil horizon and remedial measures for soil erosion. **(05 Marks)**
d. Write note on geological action of wind and river. **(06 Marks)**

4 a. What are earthquakes? Explain its causes and effects. **(06 Marks)**
b. What is Tsunamis? Explain its causes and effects. **(04 Marks)**
c. What is coastal zone and coastal land forms. **(04 Marks)**
d. What is continental shelf, continental rise, continental slope and abyssal plain. **(06 Marks)**

PART – B

5 a. What is fold? Describe any two types of folds, with neat diagram. **(06 Marks)**
b. What is fault? Classify them and describe any two types of faults, with neat diagram. **(06 Marks)**
c. What is joint? Classify the joint based on their origin. **(04 Marks)**
d. What is unconformity? Classify them. **(04 Marks)**

6 a. Up stream dipping sedimentary beds are safe for dam foundation? Discuss. **(04 Marks)**
b. Add a note on tunneling in folds. **(06 Marks)**
c. What is toposheets and geological maps. **(04 Marks)**
d. Explain how geotechnical investigation is carried out for selection of sites for dams and tunnels. **(06 Marks)**

7 a. Write a brief note on hydrological cycle. **(03 Marks)**

b. Write a note on various methods of ground water exploration. Describe in detail the electrical resistivity method of ground water exploration. **(06 Marks)**

c. What is an aquifer? Write a note on confined and unconfined aquifer and mention other types of aquifers classified based on ground water movement. **(06 Marks)**

d. Write a note on artificial recharge of ground water and rain water harvestment. **(05 Marks)**

8 a. What is remote sensing? Write its application in civil engineering. **(05 Marks)**

b. What is GPS? Write its different uses in civil engineering. **(05 Marks)**

c. Write a note on impact of mining on environment. **(05 Marks)**

d. What is landsat imageries and stereoscope and their application in civil engineering. **(05 Marks)**

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MATDIP301

Third Semester B.E. Degree Examination, December 2012

Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions.***

1 a. Find the modulus and amplitude of the complex number $1-\cos\alpha+i\sin\alpha$. **(05 Marks)**

b. If z_1 and z_2 are two complex numbers, show that $|z_1+z_2|^2+|z_1-z_2|^2=2\{|z_1|^2+|z_2|^2\}$. **(05 Marks)**

c. Find the fourth roots of $-1+i\sqrt{3}$. **(05 Marks)**

d. If $2\cos\theta = x+\frac{1}{x}$, prove that $2\cos r\theta = x^r+\frac{1}{x^r}$. **(05 Marks)**

2 a. Find the n^{th} derivative of $e^{2x}\cos^3 x$. **(07 Marks)**

b. Find the n^{th} derivative of $\frac{x}{x^2-5x+6}$. **(06 Marks)**

c. If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$. **(07 Marks)**

3 a. Find the angle between the pair of curves $r = 6\cos\theta$, $r = 2(1+\cos\theta)$. **(07 Marks)**

b. Find the pedal equation of the curve $r^2 = a^2\sin 2\theta$. **(06 Marks)**

c. Obtain the Maclaurin's series expansion of the function $\sqrt{1+\sin 2x}$. **(07 Marks)**

4 a. If $u = x^2y+y^2z+z^2x$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=(x+y+z)^2$. **(05 Marks)**

b. If $u=\tan^{-1}\left(\frac{x^3y^3}{x^3+y^3}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{3}{2}\sin 2u$. **(05 Marks)**

c. If $u = x+y+z$, $v = y+z$, $z = uvw$, find Jacobian of x, y, z with respect to u, v, w. **(05 Marks)**

d. If $z = f(x, y)$ and $x = e^u+e^{-v}$ and $y = e^{-u}-e^v$, prove that $\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}$. **(05 Marks)**

5 a. Obtain the reduction formula for $\int_0^{\pi/2}\cos^n x\,dx$ and hence evaluate $\int_0^{\pi/2}\cos^6 x\,dx$ and $\int_0^{\pi/2}\cos^9 x\,dx$. **(07 Marks)**

b. Evaluate $\int_0^1\int_{x^2}^{\sqrt{x}}xy(x+y)dydx$. **(06 Marks)**

c. Evaluate $\int_0^a\int_0^x\int_0^{x+y}e^{x+y+z}dzdydx$. **(07 Marks)**

6 a. Define Gamma and Beta functions. Show that $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta$. **(07 Marks)**

b. Prove that $\int_0^{\infty} x^2 e^{-x^4} dx \times \int_0^{\infty} e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}$. **(07 Marks)**

c. Evaluate $\int_0^1 (\log x)^6 dx$. **(06 Marks)**

7 a. Solve the equation $\frac{dy}{dx} + x\tan(y-x) = 1$. **(06 Marks)**

b. Solve $x^2 y dx - (x^3 + y^3)dy = 0$. **(07 Marks)**

c. Solve $(e^y + y\cos xy)dx + (xe^y + x\cos xy)dy = 0$. **(07 Marks)**

8 a. Solve the equation $(D^3 + 1)y = 0$, where $D = \frac{d}{dx}$. **(06 Marks)**

b. Solve the equation $(D^2 - 2D + 1)y = xe^x$. **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{2x} - \cos^2 x$. **(07 Marks)**

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